working paper
department
of economics

TRADE, GROWTH AND CAPITAL

by

Michael Bruno

Number 65 October 1970

massachusetts
institute of
technology

50 memorial drive
cambridge, mass. 02139

TRADE, GROWTH AND CAPITAL

by

Michael Bruno

Number 65

October 1970

This study was made under a faculty fellowship granted by the Ford Foundation. However, the conclusions and other statements in the paper are those of the author and not necessarily those of the Ford Foundation.

TRADE, GROWTH AND CAPITAL

I. INTRODUCTION

This paper arises from a dissatisfaction with the present framework of growth theory when applied to an open economy setting. Such an opening sentence could probably equally well be turned around to suggest dissatisfaction with the application of international trade theory to a growing economy, except that I feel I probably know even less about the latter.

Three kinds of major hurdles come to mind. The first, starting from a standard growth model for a closed economy, is the deadweight of our usual two-sector consumption-investment goods representation with its emphasis on the accumulation of one (domestic) capital good, the change (usually a fall) in the relative price of the capital good, as some state of long run bliss is approached, and a rather curious capital intensity assumption in the midst of it all. I doubt whether any of this was ever put to empiricial test, maybe it was not meant to be. A number of economists, however, have been worrying about the closed model and during the last five years various attempts[1] have been made to open up the model for trade, still basically keeping the C-I division and allowing for the specialisation in production of one of the goods. It is not clear why, once you allow trade,

---

[1] See, for example, Oniki-Uzawa [1965], Ryder [1967] and others. More recent interesting studies that come to my mind are 3-sector models as for example work by Bardhan [1969] and Teubal [1970].

that distinction should retain its previous importance. After all you produce exports or import substitutes in order to earn (or save) foreign exchange which can be put to any use so what does it matter whether these are nylon stockings or bulldozers? This brings me to the second point.

The traditional trade theory setting is one that concentrates almost always on the other extreme case in which everything is traded and the main relative price that is featuring is the international terms of trade. In practice, even the most open economies use and produce commodities that form a substantial part of the consumers' budget and that for all practical purposes are non-tradable. The tradable/non-tradable goods distinction turns out on inspection to be no less important than that between consumption and investment and moreover cuts across the latter division rather than is identical with it. The relative price of these goods i.e. the 'real' exchange rate, seems a key decision variable in the characterisation of allocation and pricing of a growing economy. This in turn brings me to the third hurdle.

Most theoretical discussions in which trade features identify equilibrium with a balance on current account. There is, of course, extensive treatment in the trade literature of short run deficits and surpluses, the role of money and of ways to correct imbalances, but there is little if any systematic treatment of prolonged structural imbalances that are a necessary by-product of development accompanied by foreign borrowing or foreign aid. There have recently been some studies which have opened up the one sector growth model to foreign borrowing.[1] However,

---

[1] But with no trade, see Bardhan [1967] and Hamada [1969].

I am not aware of any systematic dynamic theory that combines both trade and foreign borrowing. This is not to say that economists have not been aware of it - in fact any reader of the empirical planning model literature would realize that some of these aspects by now feature as central components in any systematic development planning framework. The trouble with complex empirical models, however, is that so much depends (or at least appears to depend) on particular specification, that some of the broader theoretical conclusions get completely lost in them.

In principle there should be no difficulty in suggesting a comprehensive framework in which all of these hurdles would be simultaneously overcome. The difficulty lies in finding clear and easy-to-solve simplifications thereof and that, in short, is the object of this paper - to suggest a suitable general equilibrium framework which is relatively simple and manageable, yet incorporates all of these aspects, and to obtain from it some general results, which can and to some extent have been put, to empirical validation.

As is common practice and being the two-dimensional creatures that we are, we start our analysis in the next Section with a new but essentially simple type of two-sector model for a growing economy facing imperfect capital and export markets. One good, 'foreign exchange', after considerable groundwork, is made to perform the heavy multiple duty of juggling all tradable goods, accumulating physical stocks of machines, as well as storage of value and debt. The other good is a non-tradable consumption good. Section III is an analysis of the workings of this model along a finite horizon growth and structural change path in which a government 'optimally'

handles the real exchange rate. The latter turns out, under fairly general assumptions, to exhibit monotonic behavior over time, with some clear implications for devaluation and commercial policies. Section IV, by way of digression, gives a full quantitative solution for one specification of such a model, a Cobb-Douglas world.[1]

Section V gradually introduces miscellaneous complications in the form of a non-tradable capital good, infant industries, trade restrictions and wage constraints. The last section briefly discusses policy and the possible descriptive relevance of some of the results.

---

[1] A short paper more or less based on that section was delivered at the June 1970 Far Eastern Meeting of the Econometric Society in Tokyo. I would like to mention my thanks to Professors Uzawa, Hamada, Negishi and Sakashita for comments on that paper. Thanks are also due to M. Fraenkel of the Bank of Israel for comments and discussions of that and of related issues that he and I have been working on both separately and together.

II. THE BASIC OPEN ECONOMY MODEL

We consider an open ecomony that is small enough compared to its trading partners not to affect its own import prices, but may be facing downward sloping demand curves for its own exports. Although this is not necessary we shall also assume import prices to remain constant over time. Thus we can from now on measure the quantities of all importable goods -- finished consumer goods (C_m), investment goods (I_m) and intermediate inputs (M_i) -- with the aid of the composite yardstick of foreign exchange ($) cost. The same, though requiring somewhat longer proof will be found to apply to the domestic <u>production</u> (E) of foreign exchange through heterogenous exports (or <u>saving</u> through import substitutes), a subject to be discussed in detail below. The current deficit (M-E) will be financed through foreign borrowing in an imperfect capital market.

While an importable commodity may or may not be imported in fact (see below), our economy will be assumed to produce domestically a non-tradable consumption good (C_d) as well as an investment good (I_d). A non-tradable investment good will be introduced explicitly only at a later stage (see Section V) and for the moment all investment goods will be assumed to be importable.[1] As we shall see that greatly simplifies and focuses the discussion while at the same time does not change the essential features of the problem.

The assumed existence of at least one domestic non-tradable con-

---

[1] For most developing economies, throughout much of the relevant phaze, most of equipment could in fact be assumed to be <u>imported</u>, but we don't have to go that far.

sumption good, however, is of great importance to the analysis. Moreover, this we also consider an important feature of real life. The fact is that even the most open economies domestically produce something like 75-80% of their consumption in value terms, leaving aside possible Andora or San Marino-like exceptions. You may in principle live in London, commute to New York everyday and have your hair cut on the Riviera during the weekend, but how many do? Important magnitudes such as the quantity of money, the real wage rate and the real foreign exchange rate are usually measured in reference to a commodity standard largely determined by non-traded domestic consumption goods.

Our scheme will not explicitly introduce domestic production of intermediate goods. Here we follow a useful convention of working only with final goods and primay factors of production, assuming all intermediate goods have been expressed through the indirect input of primary factors.

We have a domestic non-tradable primary factor, call it labour (L), which will be assumed to be growing exogenously and is allocated to the production of the various goods. Full employment of labour is not mandatory but where it will not occur, some minimum wage constraints will be introduced (See Section V). We now turn to a more detailed specification of the model.

1. The Net Export Revenue Function

Consider any one export industry. Assume a production function $Q(X_i; t)$ for the quantity of exports supplied, where X_i are factor inputs and t is time. The function is assumed to have positive first and negative second derivatives for each factor and to be homogenous of degree δ.

Next assume export demand to be given by the function $Q_D = P^{-\eta}Z(t)$, where P is the world price, η is the elasticity of demand, assumed to be greater than 1, otherwise it will never pay to increase input into that export industry, and Z(t) is an exogenous shift factor. For gross export revenue (E), measured in foreign exchange, we thus have:

$$(1) \qquad E = PQ = [Q(X_i;\ t)]^d\ Z(t) = E(X_i;\ t)$$

$$\text{where } d = 1 - \frac{1}{\eta} \qquad \text{and} \qquad 0<d\leqq 1$$

Consider the derived revenue function $E(X_i;\ t)$. It follows from straight differentiation of the underlying expression that

$$\frac{\partial E}{\partial X_i} > 0 \quad , \quad \frac{\partial^2 E}{\partial X_i^2} < 0$$

and from the Euler equation for homogenous functions we have:

$$\sum \frac{\partial E}{\partial X_i} X_i = d\ Q^{-\frac{1}{\eta}}\ Z(t)\ \delta Q = d\delta E$$

Hence the new function E has all the properties required of a production function, output now being foreign exchange. Its degree of homogeneity is $d\delta\leqq\delta$. We could thus have an underlying increasing returns technology and still get $d\delta\leqq 1$, and it is only the latter that we shall have to assume throughout most of our analysis. We shall also assume that the second cross-deriviatives are positive or zero.[1]

Next let us specify the input structure. Consistent with the above introductory remarks we start by considering three primary inputs, labour

---

[1] This holds automatically if $d\delta=1$.

(L_E), imports of intermediates (M_E), and a stock of imported capital (K_{ME}), both of the latter being measured in foreign exchange value terms ($), and possibly another fixed factor (A_E). We now assume that all firms either directly or indirectly face a given rate of interest rate (r) on the foreign markets. The Composite Good Theorem will allow us to aggregate both intermediate imports and the imputed services of imported capital (assuming a fixed exponential depreciation rate μ) into one composite imported input. Moreover, in view of the fact that the marginal productivity of a current foreign exchange input in its own reproduction must be unity, we can turn (1) into

$$\bar{E} = \bar{E}(L_E;\ t,\ A_E) \tag{2}$$

with $$\frac{\partial \bar{E}}{\partial L_E} = \frac{\partial E}{\partial L_E} > 0 \quad \frac{\partial^2 \bar{E}}{\partial L_E^2} \lesseqqgtr 0$$

where $\bar{E} = E - M_E - (r + \mu)K_{ME}$ = <u>net</u> export revenue and A_E = a fixed factor, say land or resources, which from now on we may ignore (see below).

One can show the assertion underlying (2) in the following form: Consider any production function $F = F(X,Y)$ for which

$$\frac{\partial F}{\partial X} = f(X,\ Y) = a \text{ (fixed parameter)}$$

and we want to express X as a function of Y and substitute back into the original function F. This we can do since

$$\frac{\partial f}{\partial X} = \frac{\partial^2 F}{\partial X^2} < 0$$

We have:

$$\frac{\partial f}{\partial X}\frac{dX}{dY} + \frac{\partial f}{\partial Y} = 0$$

or
$$\frac{dX}{dY} = -\frac{\frac{\partial f}{\partial Y}}{\frac{\partial f}{\partial X}} = -\frac{\frac{\partial^2 F}{\partial X \partial Y}}{\frac{\partial^2 F}{\partial X^2}}$$

Now introduce a new function $\bar{F}(Y)$:

$$\bar{F}(Y) = F[X(Y), Y] - aX(Y)$$

We have:

$$\frac{\partial \bar{F}}{\partial Y} = \frac{\partial F}{\partial X}\frac{dX}{dY} + \frac{\partial F}{\partial Y} - a\frac{dX}{dY} = \frac{\partial F}{\partial Y} > 0$$

$$\frac{\partial^2 \bar{F}}{\partial Y^2} = \frac{\partial^2 F}{\partial Y^2} + \frac{\partial^2 F}{\partial X \partial Y} \cdot \frac{dX}{dY} = \frac{\partial^2 F}{\partial Y^2} - \frac{\frac{\partial^2 F}{\partial X \partial Y}}{\frac{\partial^2 F}{\partial X^2}}$$

Thus
$\frac{\partial^2 \bar{F}}{\partial Y^2} < 0$ if $F(X,Y)$ has a positive Hessian i.e. shows decreasing returns to scale in the two factors X, Y (it will be zero if F shows constant returns in these factors).[1]

---

[1] This is the reason why it is preferable in the present context, to have $\bar{E}$ show decreasing returns in L_E and introduce another fixed factor (A) which earns the difference between wages (WL_E) and E. This will no longer be required when a domestic capital input is introduced explicitly.

In this way we can do away with the explicit inclusion of the capital input in E. Implicitly, however, it is still there and we now have to keep in mind that $\bar{E}$ will be a function of r and that

$$\frac{\partial \bar{E}}{\partial r} = - K_E < 0.$$

The next stage is to note that when there are many export industries each of which satisfies the set of conditions specified above, we derive a net export revenue function as in (2) for each one of them and, by a very similar aggregation procedure, form a composite export revenue function

$$E = E\ (L_E;\ t) \quad (\frac{\partial^2 E}{\partial L_E^2} < 0)$$

in which L_E will now refer to the total labour employed in the aggregate export sector.[1]

Finally let us observe that the composite net foreign exchange output (from now on denoted for simplicity by E) need not refer only to exports but will encompass foreign exchange saving through the production of import substitutes which may formally be treated the same as exports. What this implies is that whenever we speak about imports of intermediate goods (M_i), consumer goods (C_m) or capital goods ($\dot{K}_M$) what in fact will be meant is the input of tradables which may or may not be imported or produced domestically (in the "E" sector) depending on the general equilibrium solution. What concerns us here is not the absolute level of exports or actual imports (an arbitrary notion in the national accounting system anyway) but only the difference (E-M) between exports

1 This is done by maximizing $E = \sum_i E_i(L_{E_i})$ subject to $L_E = \sum_i L_{E_i}$ and expressing E as a function of L_E.

and imports in foreign exchange value terms, and that will not be affected by such a procedure.

The pleasing feature of all of these derivations is the fact that instead of having to deal with a complex many factor multi-sector trade system we can, under a fairly broad set of conditions, given heterogenous exports and non-competitive world markets, still compress the whole tradable commodity supply system into one simple aggregate relationship.[1]

2. Consumer Goods

Consumer goods in our system may take one of two forms. There is one composite finished consumer good that is importable (C_m) and therefore measured in terms of foreign currency ($) value.[2] There is a non-tradable single (or composite) consumer good that is always domestically produced and will be denoted by C_d, its production function takes the form:

(3) $$C_d = C\ (L_C, M_C;\ t)$$

where L_c is the input of the primary labour factor and M_C stands for the composite input of imports (tradables), comprising as in the case of the E-sector, both intermediate inputs as well as imported capital goods

---

[1] An empirical example using a piecewise linear model is given in Bruno [1967, 1970]. It should be pointed out, however, that the above formulation does not always take good care of real life. In particular this becomes unsatisfactory for a good for which there are considerable international transport costs and the level of profitable production of it as an import substitute will explicitly depend on the size of the domestic market. In such case import substitutes have to be distinguished from exports (of the same good) and the E sector respectively disaggregated. This presents no problem in principle. A much harder problem is the one in which one of the E functions would show truly increasing returns. (One form of externality, namely 'learning', will be discussed in Section V).

[2] See previous remark on the treatment of import substitution.

in the form $(\mu + r) K_{MC}$, say. C will be assumed to show constant or decreasing returns to scale in L_C and M_C.[1] According to our specification C_d cannot be substituted for C_m in production. We will, however, allow for substitution in consumption, by specifying a concave utility function $U(C_d, C_m)$, to be discussed below.

3. Foreign loans and the balance of payments constraint

Let us define, for our purposes, the net current account surplus (or deficit) as $F = E - (C_m + M_C)$ (this will be negative in case of a deficit). In order to get the more usual definition of a surplus one has to subtract from F (or add to the deficit) both interest payments on the net debt (denoted by R_B) as well as net investments in the form of importable capital goods $\dot{K}_M$. We denote the net outstanding foreign debt by B and its time change dB/dt by $\dot{B}$. These magnitudes will relate to the outstanding debt net after deducting the stock of tradable assets held by producers in the economy (K_M), at any point in time.[2] The same letters with a bar will relate to the gross debt ($\bar{B} = B + K_M$), which is the more familiar debt concept in balance of payments discussions. We accordingly have:

$$(4) \qquad \dot{B} = R_B - F = R_B - [E - (C_m + M_C)]$$

and

$$\dot{\bar{B}} = \dot{B} + \dot{K}_M = E - (C_m + M_C + \dot{K}_M + R_B); \qquad \dot{K}_M = -\frac{d}{dt}\left(\frac{\partial F}{\partial r}\right)$$

---

[1] As in the case of the trade sector, we must now keep in mind that both M_C and therefore C_d will be functions of the interest rate r which will be exogenous to the industry but may be endogenous to the system as a whole.

We have $\dfrac{\partial M_C}{\partial r} = K_{MC} > 0$

[2] From our previous analysis we know that $K_M = K_{ME} + K_{MC} = -\dfrac{\partial F}{\partial r}$ and the interest costs rK_M appear in the respective production functions.

The advantage of using the net debt concept in the present analysis is closely tied with the assumptions that will be made with respect to the workings of the capital market. We shall assume that the economy faces an imperfect capital market on which it can borrow long (or lend) at a marginal interest cost $r(B)$ which is an increasing function of the net outstanding debt B ($r'(B) \gtreqless 0$ and we also assume $r''(B) \gtreqless 0$). We thus have $R_B = {}_0\int^B r(x)dx$, assuming that each loan carries the interest rate at which it was taken until the date it is repayed. On the other hand, any amount of additional foreign exchange can be borrowed short at the rate $r(B)$ to finance imports of capital goods, thus the marginal interest cost on the gross debt is determined by the existing net debt. When B changes r may change accordingly. This is consistent with our assumption that producers in the two sectors face a given interest rate on their tradable assets, which is exogenous to them, but is endogenous to the system as a whole. It also implies that the tradable capital stock is assumed to be completely shiftable, once purchased it could be resold on the world market at no capital loss, other than the depreciation factor μ.[1]

What this set of assumptions means is that as long as a producer (or the economy as a whole) borrows in order to purchase capital goods foreign firms (or governments) are willing to lend the money at the going rate, since there is no extra risk involved (e.g. tied aid). Where the extra risk comes in is in loans (or investments) that do not show immediately in the form of tangible tradable assets. Also implied is an assumption that all producers (and consumers) borrow through some central channels, either the

---

[1] A non-shiftability assumption could be incorporated here, but it would obscure the main issues and besides, as long as the economy grows and accumulates K_M, such problem is unlikely to occur.

domestic government being the only representative of the borrowers *vis à vis* the rest of the world or the foreign capital market being highly centralized or both. At any rate different parts of the foreign market must view the riskiness equally, and they must all be considering the net debt position of the country as a whole. None of these assumptions seems unduly unrealistic.

4. Intertemporal Welfare and Policy Framework

Having discussed the main production and trade constraints we must not specify how our system allocates its resources at any point in time and what makes it move from one point in time to the next. For the time being we make the system run as a policy model and ask ourselves, given that the economy wants to go from a value B_0 in time 0 to B_T at time T and is maximising a simple intertemporal social welfare function what are the characteristics of such optimal behavior and what is the path of the key variables and policy instruments over time. The main instrument that the government will have at its disposal is the real exchange rate or the relative price of the E and the C sectors (to be denoted by $p_\$$), and an important parameter that it will set itself is the pure social rate of discount (q). We shall subsequently deal with other policy formulations and with an alternative interpretation of the system as a descriptive model of behavior in an (internally) competitive environment. The simplest kind and at the same time reasonably general formulation of our criterion will be to assume

$$\text{(5)} \qquad \text{Max} \int_0^T U(C_d, C_m)\, e^{-qt}\, dt$$

with $q>0$, and $U_{ij}<0 \quad i = j$

$U_{ij} \gtreqless 0 \quad i \neq j$ (a less stringent assumption can be made)

A more exact specification of U as well as osme alternative specifications of the maximand, will be discussed as we go along.

As is well known from optimal growth theory the time additivity of welfare is a source of great simplification, for as the use of the calculus of variations, dynamic programming or the Pontryagin maximum principle[1] will tell us our system can now be decomposed into the sequential solution of the following instantaneous maximization problem:

(6) $$\text{Maximize } H = [U(C_d, C_m) - \pi_\$ \dot{B}] \quad (0 \leqq t \leqq T)$$

subject to:

(7) $L_c + L_E \leqq L(t)$ (full employment constraint)

(2) $E = E(L_E; t)$ (foreign exchange revenue function)

(3) $C_d = C(L_c, M_c; t)$ (production function for consumer goods)

(4) $\dot{B} = R_B - [E - (C_M + M_c)]$ (balance of payments and foreign borrowing)

where $B(0) = B_0, \quad B(T) = B_T$ given

$(R'_B = r(B), \; r'(B) \gtreqless 0, \; r''(B) \gtreqless 0)$

Finally $\pi_\$$ the (undiscounted) utility price of foreign exchange must satisfy the Euler equation

$$\frac{d(\pi_\$ e^{-qt})}{dt} = - \frac{\partial H}{\partial B} e^{-qt}$$

---

[1] For references on the latter see, for example, Arrow and Kurz [1970] or Shell [1967].

which, when worked out from (6) and (4) gives

$$(8) \qquad \frac{\dot{\pi}_\$}{\pi_\$} = q - [r(B) - \frac{\partial F}{\partial r} r'(B)]$$

The latter is a relationship familiar from many capital (and growth) models: The imputed utility price of foreign exchange must change over time at a rate that equals the difference between the pure rate of discount q (the marginal rate of substitution of present and future utilities) and between the marginal cost of foreign borrowing.[1] Looked at from a different point of view all that (8) says is that the marginal productivity of foreign exchange (or own rate of interest) $(q - \dot{\pi}_\$/\pi_\$)$ must equal the marginal social cost of foreign borrowing. Below we shall express it in yet another form in terms of the real exchange rate

$$p_\$ (= \frac{\pi_\$}{U_1})$$

Let us also note that H can be written in the form:

$$(9) \qquad H = [U - (U_1 C_d + U_2 C_m)] + [(U_1 C_d + U_2 C_m) + \pi_\$ (E - M - R_B)]$$

U_1 and U_2 are the respective marginal utilities.

The first square brackets represents the consumers' surplus, which in the case of a linearly homogenous utility function will be zero, whereas the second square brackets represents the net national product (in terms of utility units). As will be clear from our subsequent discussion maximizing H (subject to the various constraints) is tantamount to max-

---

[1] Note that when B changes the cost of holding K_M also rises, by

$$K_M r'(B) = - \frac{\partial F}{\partial r} r'(B)$$

imizing the national product with prices equal their marginal utilities. We could thus reinterpret our system as one in which the economy at each point of time is assumed to maximize national income (product)[1] and prices behave in a way that equalizes the rate of return on tradable assets in all their uses.

---

[1] The fact that explicit investment is missing from the present expression for NNP is only due to the assumption the $I_d = 0$. Note that we could add $\pi_\$ \dot{K}_M$ to consumption and subtract the same term in the net export term as an import item to make the second brackets in H look more familiar.

III. THE WORKINGS OF THE MODEL

Let us now characterize some solutions of the model outlined in the previous section. If we differentiate the lagrangian expression $[H + \omega(L-L_C-L_E)]$ with respect to L_C, L_E, and M_C and C_m, equating to zero and further introduce the notation

$$p_\$ = \pi_\$/U_1 \qquad w = \omega/U_1 \qquad \text{(on the assumption } U_1>0)$$ [1]

we get:[2]

$$(10) \qquad \frac{\partial C}{\partial L_C} = p_\$. \frac{\partial E}{\partial L_E} = w \qquad [L = L_C + L_E]^3$$

$$\text{and} \quad (11) \qquad \frac{\partial C}{\partial M_C} = p_\$ = \frac{U_2}{U_1}$$

Let us introduce the following notation: write C_1, C_2 (and similarly E_1) for the partial derivatives of C with respect to L_C, M_C respectively (and similarly for E) and likewise $C_{ij}(i, j=1, 2)$ and E_{11} for the various second derivatives.

The relevant part of the system (10) - (11) can be written in the form:

$$F_1: \qquad C_1 - p_\$E_1 = 0$$

$$F_2: \qquad C_2 - p_\$ \qquad = 0$$

$$F_3: \qquad L_C + L_E - L(t) = 0$$

---

[1] The case $U_1 = 0$ will be mentioned briefly below.

[2] These are necessary conditions for a maximum. Sufficiency is assured through the concavity of the maximand.

[3] The possibility $L_C + L_E < L$ (and $w = 0$) will be discussed in Section V. Here we assume the technology allows for sufficient substitution so that this does not happen.

This leads to the following Jacobian with respect to the four variables L_C, L_E, M_C and $p_\$$:

$$(12) \qquad \frac{\partial(F_1, F_2, F_3)}{\partial(L_C, L_E, M_C, p_\$)} = \begin{bmatrix} C_{11} & -p_\$E_{11} & C_{12} & -E_1 \\ C_{12} & 0 & C_{22} & -1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

We have $C_{11}, C_{22}, E_{11}<0$, $E_1>0$.

Let us also assume that

$$C_{12} \geqq 0$$

and that the Hessian of the C function is non-negative i.e.

$$H_{12} = C_{11}C_{22} - C_{12}^2 \geqq 0 .$$

The case $H_{12} = 0$ is the one in which C shows constant returns in its two arguments, and the case of strictly positive H_{12} is the one of decreasing returns (we do not allow increasing returns).[1]

Denoting the determinant of the first three columns of (12) by A we have $A = -p_\$E_{11}C_{22} - H_{12} < 0$. If we leave time constant and solve the system for the derivatives of the inputs with respect to the price $p_\$$ we find:

$$(13) \qquad \left[\frac{dL_C}{dp_\$}\right]_t = -\frac{C_{22}E_1 + C_{12}}{A} < 0 \qquad \left[\frac{dM_C}{dp_\$}\right]_t = -\frac{p_\$E_{11} - C_{11} + C_{12}E_1}{A} < 0$$

---

[1] If C is constant returns we automatically have $H_{12} = 0$ and $C_{12} \geqq 0$. To obtain the forthcoming results, however, it is not necessary to assume $C_{12} \geqq 0$ and a slightly less restrictive assumption can be made.

and $$\left[\frac{dL_E}{dp_\$}\right]_t = \frac{C_{22}E_1 - C_{12}}{A} > 0$$

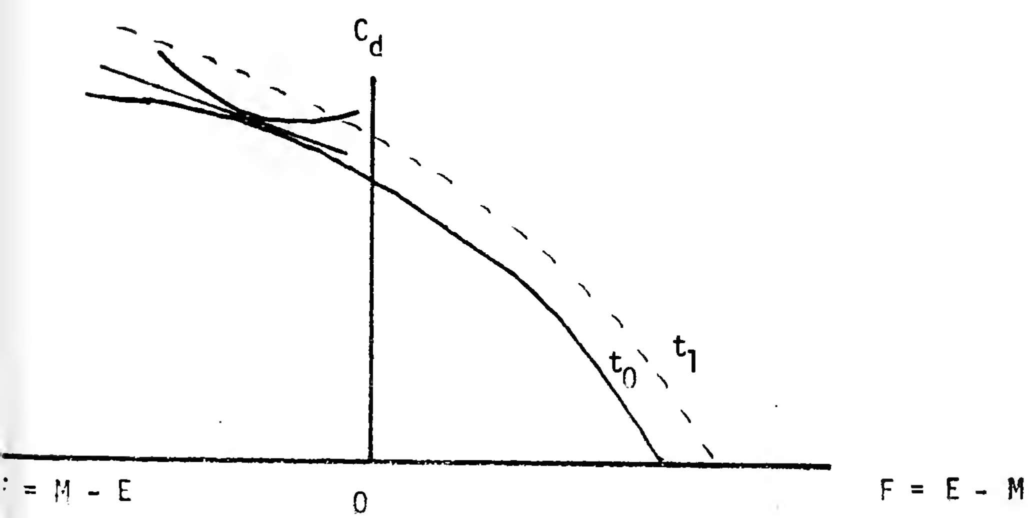
It follows that $\left[\frac{dC_d}{dp_\$}\right]_t < 0$ and $\left[\frac{dE}{dp_\$}\right]_t > 0$

Next, if apply a similar notation to the utility function and use the second equation of (11) we get, on the assumption that $U_{12} \gtreqless 0$ and U_{11}, $U_{22} < 0$:

$$(14) \qquad \left[\frac{dC_m}{dp_\$}\right]_t = \frac{U_1 + \left[\frac{dCd}{dp_\$}\right]_t (p_\$ U_{11} - U_{12})}{U_{22} - p_\$ U_{12}} < 0$$

Putting together the various ingredients of the balance of payments surplus $F = E - (C_m + M_C)$ we find that $\left[\frac{dF}{dp_\$}\right]_t > 0$, a 'real' devaluation increases the surplus or, rather, decreases the deficit. What all this leads up to is that we can describe our economy at any point in time in terms of two functions $C_d = C(p_\$; t)$ and $F = F(p_\$; r, t)$ or a production possibility curve linking F and C_d having the usual concavity properties, with its slope at any point giving the value of $(-p_\$)$ (see figure 1) :

Figure 1



What makes this PPC different from the usual closed economy two-sector representation is the fact that one of the goods (F) can be negative. In fact it is the left hand quadrant that is the relevant one during net borrowing periods.

Having obtained a reduced form of the static system we can now turn back to the dynamic behavior equations (4) and (8). At this point it will simplify matters if we choose a specific, though sufficiently general, form for the utility function:

$$(15) \qquad U(C_d, C_m) = C_d^{\alpha} C_m^{\beta} \qquad (0<\alpha \leq 1-\beta, \quad 0 \leq \beta < 1)$$

We then get in equation (11)

$$p_\$ = \frac{U_2}{U_1} = \frac{\beta}{\alpha} \cdot \frac{C_d}{C_m}$$

and the price adjustment equation (8) will take the form:

$$(8') \qquad \frac{\dot{p}_\$}{p_\$} = \frac{1}{1-\beta} \left[q + (1-\alpha-\beta) \frac{\dot{C}d}{Cd} - \bar{r} \right]$$

where $\bar{r} = r - \frac{\partial F}{\partial r} r'$

A number of special cases deserve mention. First, when $\alpha + \beta = 1$, the term involving the rate of change of C_d in (8') drops out and we are left with the simple equation:

$$\frac{\dot{p}_\$}{p_\$} = \frac{1}{1-\beta} \left[q - \bar{r} \right].$$

Next, the case $\beta = 0$, in which the traded good does not appear in the utility function, gives an equation which is familiar from optimal growth theory for the closed economy. We have excluded the case $\alpha = 0$ (no domestic

good) here and it may at this stage be worthwhile to point out what the implications would have been.

When there is no domestic non-tradable good in the system, the marginal utility of consumption (U_2) must equal the nominal exchange rate ($\pi_{\$}$) at all times, there is only one commodity (\$) in the system and there is no sense in which we can then speak of a change in the real exchange rate.[1] This points out where the importance of realistically distinguishing between tradables and non-tradables lies in any trade and growth theory.

Consider now the system of dynamic equations (4) and (8') written in the following form:

$$\dot{B} = R_B - F[p_{\$};\, t,\, r(B)] \tag{16}$$

and

$$\frac{\dot{p}_{\$}}{p_{\$}} = \frac{q + (1-\alpha-\beta)\,\frac{1}{C}\frac{\partial C}{\partial t} - [\, r(B) - \frac{\partial F}{\partial r}\, r'(B)]}{(1-\beta) - (1-\alpha-\beta)\,\frac{p_{\$}}{C}\,\frac{\partial C}{\partial p_{\$}}}$$

It will help to reiterate at this point what the expected direction of change of the various components are and also add two more assumptions. Looking at (16), R_B is an increasing function of B and F is an increasing function of $p_{\$}$ and (usually) also of t. We have already noted that

$$\frac{\partial F}{\partial r} = -K_M < 0, \text{ thus } -\frac{\partial F}{\partial B} = -\frac{\partial F}{\partial r}\, r'(B) \gtreqless 0.$$

---

[1] Note that the real exchange rate in our model is always defined per unit of nominal foreign exchange (\$). Had we defined it per unit of real quantities of foreign goods then this rate might, of course, change here too but only to the extent there is a change in the international terms of trade.

We shall now make a reasonable assumption about the second cross derivative of F:

$$\frac{\partial^2 F}{\partial r \partial p_\$} = - \frac{\partial K_M}{\partial p_\$} \leqq 0.$$

The latter means that, other things being equal, when $p_\$$ rises, i.e. when E rises and C falls the total capital requirements cannot fall. This means that the trade sector must have a higher capital-output ratio than the domestic consumption sector. This is not a crucial assumption at all but it is realistic and making it absolves us from having to look into a number of subcases. Correspondingly we will need another assumption

$$\frac{\partial^2 F}{\partial r^2} = - \frac{\partial K_M}{\partial r} < \frac{dB}{dr},$$

i.e. the response of the capital stock input K_M to changes in the interest rate must be smaller that the response of the external debt, otherwise as B falls K_M will rise by more and the <u>gross</u> debt will never fall.

In (17) C (shorthand for C_d)is a decreasing function of $p_\$$ and (usually) an increasing function of time, thus the time rate of change

$$\frac{1}{C} \frac{\partial C}{\partial t},$$

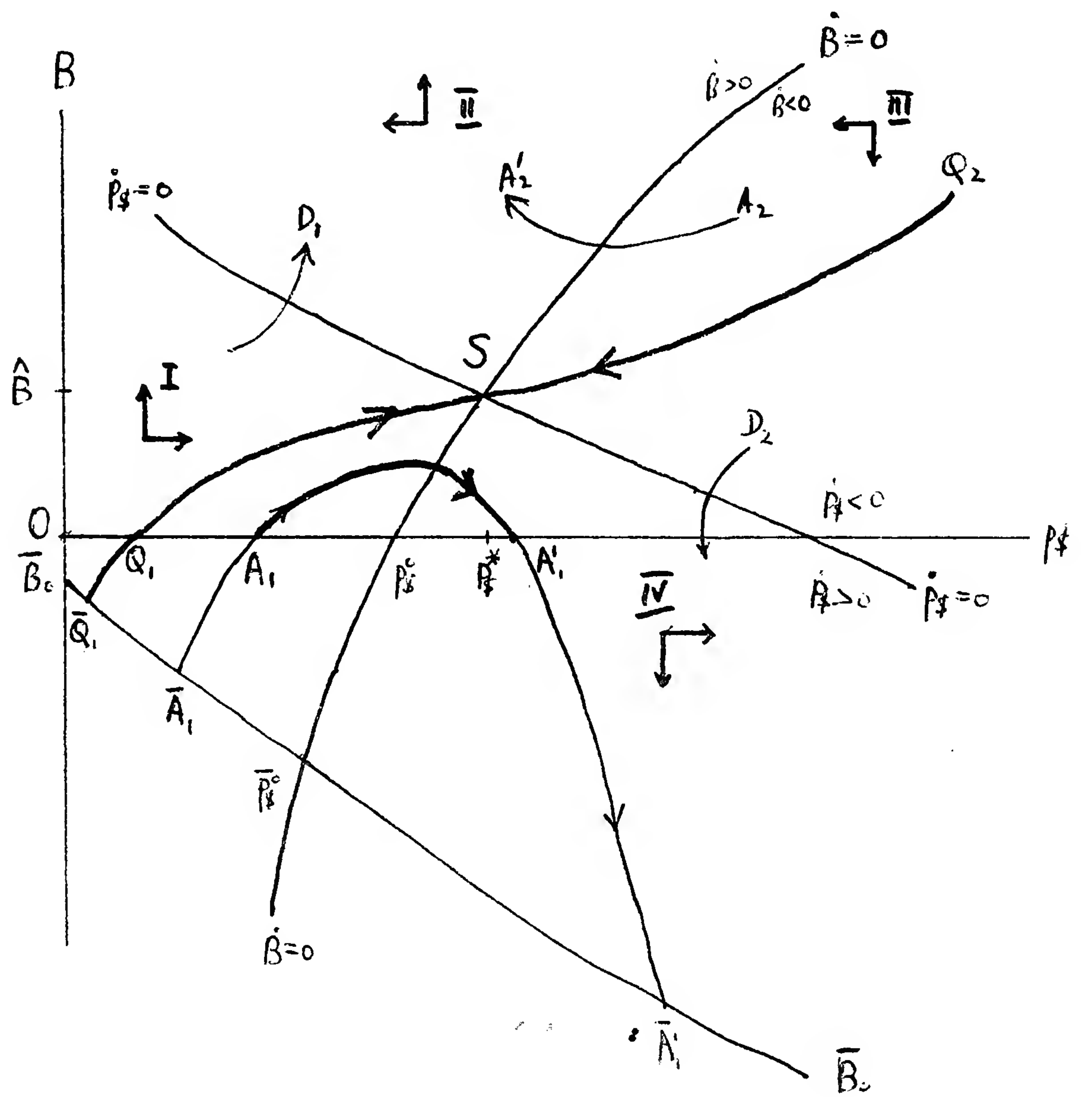
moving from one PPC in figure 1 to the 'next' at parallel slopes, will be positive. On the other hand,

$$\frac{p_\$}{C} \frac{\partial C}{\partial p_\$},$$

appearing in the denominator of (17), is the elasticity of C with respect to changes in $p_\$$ and is negative, thus all of the denominator of (17) is positive.

To get an idea about the workings of the system it will pay to consider the autonomous (timeless) case first. This will occur when time does not appear explicitly in the F function (say no population growth and no technical progress) and either t does not appear in the C function or appears only in a Hicksian exponential factor form or else $\alpha+\beta=1$, in which case the expressions in (17) are free of time. The phaze diagram of the system then looks as in figure 2:

Figure 2



The B-$p_\$$ space is divided up by the two stationary lines ($\dot{B} = 0$ and $\dot{p}_\$ = 0$) into four areas with unambiguous directions of movement for the various variables when out of the stationary state.

The $\dot{B} = 0$ line rises from left to right (the slope being $\frac{1}{r}\ \frac{\partial F}{\partial p_\$}$ which is positive) cutting the $p_\$$ axis at the point $p^o_\$$ which represents the exchange rate corresponding to complete balance on current account when the net debt is zero. To the left of this line $\dot{B}>0$ and B is rising and to the right B must be falling (this follows from the fact that $-\partial F/\partial p_\$$ is negative).

In principle there are a number of cases that might arise for the $\dot{p}_\$ = 0$ line. It may as in the simple case $r'(B) = 0$ not exist at all (see example in the next section). If

$$\frac{\partial F}{\partial r} = 0 \quad \text{or} \quad \frac{\partial^2 F}{\partial r \partial p_\$} = 0$$

it would be a horizontal line. In all other cases its slope

$$\frac{\partial^2 F}{\partial r \partial p_\$} \bigg/ \left(1 - \frac{\partial F}{\partial r}\ \frac{r''(B)}{r'(B)}\right)$$

will be negative, by assumption.[1] In any case for all points above the line $\dot{p}_\$<0$ and below it $\dot{p}_\$>0$. Since for some purposes it may be more useful to look at the gross debt we have drawn curve $\bar{B}_o\bar{B}_o$ which is the locus

---

[1] We have $\frac{\partial F}{\partial r} <0$, $r'(B) >0$, $r''(B) \gtreqless 0$. This is the point at which an alternative assumption $\frac{\partial^2 F}{\partial r \partial p_\$} > 0$ would require discussion of additional forms of figure 2, none of which would alter the essential feature of the solution, however.

of zero gross debt points $B + K_M = B \frac{\partial F}{\partial r} = 0$ it must lie in the lower right hand quadrant and its slope

$$\frac{\partial^2 F}{\partial r \partial p_\$}$$

is negative by assumption and will always be less than or equal to the slope of the $\dot{p}_\$ = 0$ line at any given $p_\$$.

It is clear that the point $S(p^*_\$, \hat{B})$ which is the stationary state solution for both variables, is a saddle point, just like in so many other capital and growth models. What this means is that given an initial value of B_0 there is only one unique initial value of $p_\$$ which will eventually guide the system to the stationary state in which both the balance of payments is in equilibrium and the exchange rate will stop moving at the same time. The locus of such points is given along the line Q_1SQ_2. There is an <u>inherent instability</u> in the system and moreover, we can never be at equilibrium in one of the variables and stay there unless we already happen to be in S.[1]

However, the interest in this type of system does not necessarily derive from its long run steady state behavior but rather from its characteristic path over <u>finite</u> horizons in which an economy is allowed to borrow. A typical path of this kind is given by the curve A_1A_1' or $\overline{A}_1A_1A_1'\overline{A}_1'$ in figure 2. Given that the economy starts and must also end, say, with no net external debt i.e. on the $p_\$$ axis, there is only one general kind of path that can be taken. The initial exchange rate must lie between Q_1 and $p^0_\$$

[1] The fact that $p^*_\$$ at S is different from $p^0_\$$ comes from the fact that the economy in the former case also has to finance (or receive) interest payments on $\hat{B}$.

both B and $p_\$$ will rise until B reaches a maximum ($\dot{B} = 0$) which is unique[1], after which the economy will embark on a balance of payments surplus ($\dot{B} < 0$), starting to reduce its external debt until the $p_\$$ axis ($B = 0$) is reached again.

Because of our treatment of importable capital goods like foreign exchange assets it might be more reasonable to assume the economy to end up at a point at which the gross debt is zero, i.e. on the $\bar{B}_o\bar{B}_o$ line. No essential qualities of the above solution are changed thereby,[2] only the quantitative relationship between $p_\$$ and T. Probably the most interesting result is the fact that throughout the process $p_\$$ must be continuously rising.

The polar case is given by curve $A_2A'_2$ for the less likely situation in which the optimal policy is to accumulate reserves[3] up to a point and then run them down. In either case $p_\$$ must show monotonic behavior throughout the growth path, and these are the only two cases possible.[4]

Suppose the government makes errors and is not really able to know exactly where its initial $p_\$$ at A_1 (or at $\bar{A}_1$) should be. As long as it sets $p_\$$ somewhere between Q_1 and $p^o_\$$ (or $\bar{Q}_1$, $\bar{p}^o_\$$) and the economy follows

---

[1] This uniqueness critically depends on the assumption that $r'(B) \geqq 0$, otherwise there might be multiple equilibria.

[2] It is here that we make use of the assumption $\frac{\partial^2 F}{\partial r^2} < \frac{dB}{dr}$ because that will ensure that $\dot{\bar{B}} = 0$ will also have a solution.

[3] This will occur e.g. when the initial $r(B_o)$ lies above the social rate of return and the line $\dot{p}_\$ = 0$ lies below $\bar{B}_o\bar{B}_o$.

[4] The other dual pair of curves D_1 and D_2 shown in the figure, where $p_\$$ first rises then falls or vice versa, apply to cases where B behaves monotonically throughout. This, in the present context, cannot occur but may arise in a case of non-optimal behavior. (see Section VI).

the rules of dynamic behaviour set by the pair of equations (16) - (17), the general qualitative behavior of the economy will remain the same as depicted above, except that the time T taken to reach back to the zero debt position will be different (undoubtedly longer, usually...) however, $p_\$$ must rise in the process. It is on the basis of such considerations that we believe there is a great deal of relevance in this bype of analysis for devising practical policy rules.

Once we allow time in explicitly, it is harder to see what happens but it seems that the general qualitative features for the finite horizon case remain. The curve $\dot{B} = 0$ in terms of figure 2 will now move to the left. The other curve, $\dot{p}_\$ = 0$, will shift upward. Thus what will change are essentially only speeds of adjustment, but not the general directions. It is hard to give any general results in this non-automous case unless one spells out some of the production relations more explicitly. We therefore digress in the next section to illustrate a complete solution of one such model, namely Cobb-Douglas functions with neutral technical progress under the assumption that r(B) is constant.

IV. DIGRESSION: AN ILLUSTRATIVE TWO-SECTOR EXAMPLE

Consider now the simple example of an economy starting with an initial net debt B_o, planning its production over a finite horizon T, under the constraint that it can borrow as much as it wants during the planning period at a given rate of interest r, which is fixed (i.e. $r'(B) = 0$), providing the <u>net</u> additional debt over the whole period T is zero, i.e. $e^{-rT} B_T = B_o$.[1] For simplicity assume that the only imports are inputs into production (i.e. $C_M = 0$) and the utility function is linear in consumption. The production system is given by a simple set of Cobb-Douglas functions with technical progress. Let us first consider the production side. For the trade sector we have:

$$(18) \qquad E = b_o\, L_E^b\, e^{gt} \qquad (0 < b < 1,\ g > 0)$$

where g is a time shift factor.[2]

To avoid non-constant returns problems we can think of b_o as including also a fixed factor of production, land or entrepreneurial skills (with exponent 1-b) which earns the difference between E and the wage bill in the trade industry.

The second sector in our economy produces consumer goods C and uses labour (L_c) and foreign exchange (imports) M_c:

---

[1] The latter assumption is not necessary but simplifies the exact analytical solution.

[2] In the analysis leading to equation (1), section II, if we have one export good we might write $g = g'(1 - \frac{1}{\eta}) + z$, where g' is the 'true' technical progress factor, η is the demand elasticity and z is an exponential demand shift factor.

(19) $$C = a_o L_c^a M_c^{1-a} e^{ht} \qquad (0 < a < 1 \text{ , } h > 0)$$

Here we have constant returns and the technical progress factor is h. We note again that M_c may be interpreted to include the rental flow cost of imported capital goods (similarly for the E sector, the output being net of such capital charges).[1]

The two supply constraints for labour and foreign exchange areggiven by the following equations:

(20) $$L_E + L_c = L_o e^{nt}$$

(21) $$M_c - E = \dot{B} - r B$$

We have:

(22)
$$p_\$ = \frac{\partial C}{\partial M_c} = a_o(1-a) \left(\frac{L_c}{M_c}\right)^a e^{ht} = \frac{C(1-a)}{M_c}$$

$$w_c = \frac{\partial C}{\partial L_c} = a_o a \left(\frac{M_c}{L_c}\right)^{1-a} e^{ht} = \frac{Ca}{L_c}$$

(22) continued

---

[1] We should think of (18) and (19) as reduced forms of Cobb-Douglas functions in which originally K_M also appeared and is now expressed indirectly through an interest rate (r) expression that is submerged in the constants a_o and b_0 E.g., it is easy to show that if the original E function takes the form

$E' = \beta_o L^\beta m^{\gamma_1} K^{\gamma_2} e^{\theta t}$ the reduced form <u>net</u> E will take the form

$$\text{net } E = E' - m - (\mu+r)\, K = \frac{\beta_o}{\gamma} [\gamma_1^{\gamma_1} \gamma_2^{\gamma_2}]^{\frac{1}{\gamma}} \cdot (\mu+r)^{-\frac{\gamma_2}{\gamma}} \cdot L^{\frac{\beta}{\gamma}} e^{\frac{\theta}{\gamma}t}$$

where $\gamma = 1 - \gamma_1 - \gamma_2$.

$$w_\$ = \frac{\partial E}{\partial L_E} = b_o b\, L_E^{b-1}\, e^{gt} = \frac{bE}{L_E}$$

(22)

$$\text{and, since } \frac{w_c}{w_\$} = p_\$, \quad \frac{M_c}{L_c} = \frac{E}{L_E} \frac{(1-a)}{a} b$$

We thus get by suitable substitution

(23) $$p_\$ = [a_o a^a (1-a)^{1-a}\, b^{-a}\, b_o^{\;1/b}]\, E^{\frac{a(1-b)}{b}}\, e^{(h-\frac{ag}{b})t}$$

and by logarithmic differentiation of (23)

(24) $$\frac{\dot{p}_\$}{p_\$} = \frac{a(1-b)}{b} \frac{\dot{E}}{E} + (h - \frac{ag}{b})$$

Obviously, since $0 < b < 1$ the higher the rate of growth of exports $(\frac{\dot{E}}{E})$ the faster will be the required rate of change of $p_\$$ because the faster will the marginal rate of substitution between C and E have to change over time. When $\frac{\dot{E}}{E} = 0$ the exchange rate will rise or fall depending on the difference of the weighted productivity growth rates of the two sectors. In general,

$$\frac{\dot{p}_\$}{p_\$} \gtrless 0 \text{ if and only if } \frac{\dot{E}}{E} \gtrless \frac{ag - bh}{a(1-b)}$$

To give this condition some measure of realism consider the following fictitious empirical illustration (however orders of magnitude are more or less relevant to the Israeli economy):

$$a = 0.6 \qquad b = 0.75$$

$$g = h = 0.04$$

We have: $$\frac{\dot{p}_\$}{p_\$} = 0.2 \;\frac{\dot{E}}{E} + 0.008$$

and $\frac{\dot{p}_\$}{p_\$} > 0$ as long as $\frac{\dot{E}}{E} > -4\%$.

In fact, in the Israeli context there is an order of magnitude of $\frac{\dot{E}}{E} = 15\%$ and the above simplified set of parameters would then imply $\frac{\dot{p}_\$}{p_\$} = 3.8\%$.

Going back to the context of our optimisation model, whether $\dot{p}_\$/p_\$$ should in fact be rising over time and by how much will, of course, depend on the complete set of equilibrium conditions involving also the schedule of foreign borrowing as well as the assumptions about the utility function.[1] Let us only note here that a rising real (shadow) exchange rate is something that from the production side should not at all be unexpected.

Along with the behaviour of the exchange rate it is interesting to see the rules governing the behaviour of the wage rate over time. In a world in which the real foreign exchange rate changes over time one must distinguish between two concepts of the real wage rate--w_c and $w_\$$ as above defined. We have:

$$\frac{\dot{w}_c}{w_c} = \frac{1}{a}\left[h - (1-a)\frac{\dot{p}_\$}{p_\$}\right] \tag{25}$$

$$\frac{\dot{w}_\$}{w_\$} = \frac{\dot{w}_c}{w_c} - \frac{\dot{p}_\$}{p_\$} = \frac{1}{a}\left(h - \frac{\dot{p}_\$}{p_\$}\right)$$

Again, in the context of the above numerical illustration we might have the real wage in terms of real domestic resources rising at 4.1% per annum while the wage in terms of the dollars it earns would rise at only

---

1 E.g., if, for example, the internal consumption own rate of return is 10% and the marginal cost of foreign borrowing is 6%, the difference of 4% would be consistent with the above empirical illustration.

0.3% over time. I believe this is a distinction that is often overlooked in discussions of factor pricing and development policy in an open economy.

Turning now to the utility price adjustment equation--this will here take the simple form:[1]

$$(26) \qquad \frac{\dot{p}_\$}{p_\$} = q - r$$

which, together with (24), leads to an exponential solution for E:

$$(27) \qquad E = E_o \, e^{vt}$$

where $v = \frac{(q-r-h)b + ag}{a(1-b)}$ = given constant[2]

and E_o is an integration constant to be determined below.

In a more general case discussed in the previous section (put in equation (8') $\alpha < 1 \quad \beta = 0$) we would have:

$$(28) \qquad \frac{\dot{E}}{E} = v + \frac{b}{a(1-b)} (1-\alpha) \frac{\dot{C}}{C} \quad \text{which is} > v \text{ if } \frac{\dot{C}}{C} > 0$$

In other words, the more rapidly the marginal utility of consumption falls as C rises, the higher must be the optimal export growth rate (and, as we shall see below, the lower will be the optimal level of E_o and the higher is C_o).

Let us return to the simple exponential case and substitute for E and

---

1 Note that here, except by fluke, we cannot have $\dot{p}_\$ = 0$ (the line $\dot{p}_\$ = 0$ in Figure 2 does not exist) and if q > r, $p_\$$ will be rising throughout T. Last section we showed that this is also true for the more general case.

2 E.g., if q=10%, r=6%, h=g=0.04, a=0.6, b=0.75, we get v=16%.

M_c into the differential equation for B (21). We get:

$$(29) \qquad Y_o e^{yt} - V_o e^{vt} = \dot{B} - rB$$

where

$$Y_o = \frac{(1-a)}{a} b\, b_o^{1/b} L_o E_o^{-\frac{(1-b)}{b}}$$

$$V_o = \frac{(1-a)b+a}{a} E_o$$

$$y = \frac{an+r+h-q}{a}$$

The solution of this equation gives

$$(30) \qquad B = Z_o e^{rt} + \frac{Y_o}{y-r} e^{yt} - \frac{V_o}{v-r} e^{vt}$$

Substitution of the boundary conditions $B_0 = B_T \cdot e^{-rT}$ at time t = 0, T, respectively, gives the following values for the two constants of integration:

$$(31) \qquad \begin{aligned} E_o^{1/b} &= \frac{(1-a)b\, b_o^{1/a} L_o}{1+a(1-b)} \frac{[e^{(y-r)T}-1]}{[e^{(v-r)T}-1]} \frac{(v-r)}{(y-r)} \\ Z_o &= B_o - \frac{Y_o}{y-r} + \frac{V_o}{v-r} \end{aligned}$$

The case that is interesting is the one in which it pays to start borrowing, i.e. $Y_o > V_o$. It can be shown that this is so if and only if $v > y$.[1] We must also assume $v > r$.

In that case the economy starts at time t = 0 from a position in

---

[1] The proof of this hinges on the use of the fact that $\frac{e^x-1}{x}$ is a rising function of x, since it then follows that the product of the two expressions on the right of (31) is less than unity if and only if $v-r > y-r$ or $v > y$.

which $\dot{B}-rB > 0$, i.e. there is a net capital inflow, B rises gradually, reaches a maximum ($\dot{B}=0$) and then falls steadily ($\dot{B} < 0$) until at time T, $B(T) = B_o\ r^{rT}$ and all discounted net debts accumulated over the planning period have been repaid.[1]

Sensitivity Analysis

Let us now see how E_o [and thus correspondingly $p_\$(0)$ and inversely C(0)] varies with changes in the various given parameters:

A. $\frac{\partial E_o}{\partial T} < 0$.[2] The longer the time horizon given for the repayment of debts the lower need the initial export level be.

Next consider all the parameters appearing in (v-r) and (y-r). Suppose we are looking for sensitivity with respect to any parameter f. We have

(32) $$\frac{1}{B}\frac{\partial(\ln E_o)}{\partial f} = \frac{\partial x_1}{\partial f} F(x_1) - \frac{\partial x_2}{\partial f} F(x_2)$$

where $x_1 = v-r \quad x_2 = y-r$

and $F(x) = \frac{1}{x} - \frac{e^{xT}\cdot T}{e^{xT}-1}$

F(x) has the following properties:

$$F(x) > 0 \quad \text{for} \quad x < 0$$
$$< 0 \quad \text{for} \quad x > 0$$

---

[1] There is no special problem involved for the solution of a more general case in which we do not assume $B_T = e^{rT}B_o$, except that now E_o cannot be solved explicitly. However, one can still show that
$\frac{\partial E_o}{\partial B_o} > 0 \quad \frac{\partial E_o}{\partial B_T} < 0$ in that case.

[2] This one shows by differentiating logarithmically the function $E_o^{1/B}$ (see (31)) and this time using the fact that $\frac{xe^x}{e^x-1}$ is a monotonically increasing function of x as well as the fact that v-r > y-r.

and $F'(x) < 0$, i.e. $F(x_2) \lesseqgtr F(x_1)$ for $x_2 \gtreqless x_1$

Thus $F(x_1) < F(x_2) < 0$ for $v > y > r$

$F(x_1) < 0 < F(x_2)$ for $v > r > y$

Using these properties we find the following directions of sensitivity[1] for the case <u>$v > y > r$</u>:

B. $\frac{\partial E_o}{\partial q} < 0$

C. $\frac{\partial E_o}{\partial r} > 0$ (providing $b > \frac{1}{2}$, which is likely)

D. $\frac{\partial E_o}{\partial h} > 0$

E. $\frac{\partial E_o}{\partial g} < 0$

F. $\frac{\partial E_o}{\partial a} > 0$

G. $\frac{\partial E_o}{\partial b} < 0$

H. $\frac{\partial E_o}{\partial n} > 0$

<u>When $v > r > y$</u>: E and G always remain correct. B, D and F remain correct providing $\frac{F(x_1)}{F(x_2)} < -\frac{(1-b)}{b}$ (which is likely) but will be reversed if $b < \frac{1}{2}$ (which is unlikely). C will remain correct if $b > \frac{1}{2}$ (which is likely). H will be exactly reversed in sign in this case.

On the whole, one can say that those factors that positively affect

---

[1] The actual value of $\frac{\partial E_o}{\partial f}$ for each case can be worked out directly from formula (32).

the rate of growth of exports (v) will negatively affect the optimal initial export level, as would be expected. Cases B and C also give some indication of how our results would be affected if we turned from our specific model to the more general case discussed above. E.g., an increase in $(1-\alpha)$ (the elasticity of the marginal utility schedule) from zero will affect E_0 negatively (and C(0) positively). Similarly, a rise in the foreign borrowing rate (as B rises) will make for less borrowing, will raise the initial export (or import substitution) level and of $p_\$(0)$ and thus require a decrease in initial consumption.

V. MORE GENERAL MODELS

1. Domestic Capital Goods

We can now consider a number of possible generalizations of the model presented and analyzed in sections II and III. The obvious first candidate is to do well on our promise and introduce a non-tradable capital good(K) with corresponding gross investment (I) and depreciation rate (ε).
We get an added dynamic equation:

$$\dot{K} = -\varepsilon K + I \text{ (and } K_0 \text{ given)} \qquad (32)$$

One way of going about the analysis now is to add a third production function for I in which labour (L_I), imports (M_I) and may be the domestic capital itself (K_I) appear as inputs, similarly add capital inputs (K_C, K_E) to the two existing sectors' production functions. Alternatively we could move straight to an aggregation in the form of a three dimensional concave production possibility surface (see figure 3) $C = C(F, I; K, L; t, r)$ whose slopes must now be measured by <u>two</u> relative prices,

$$p_\$ = -\frac{\partial C}{\partial F},$$

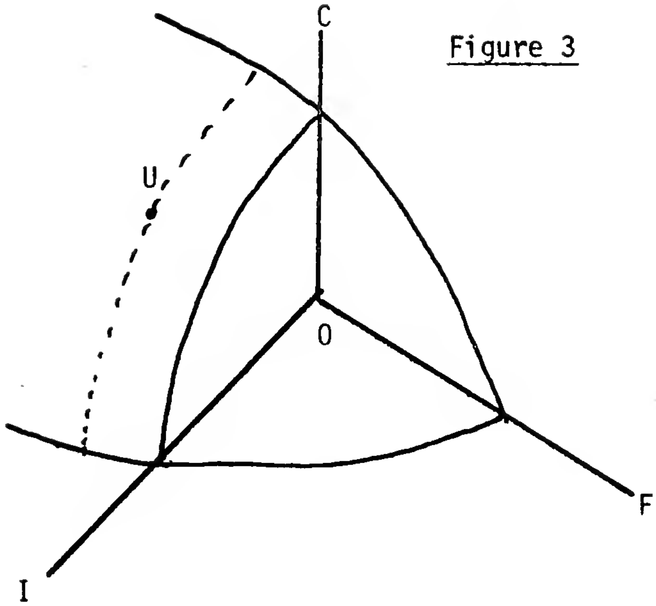
as before, and the relative price of the non-tradable investment good $p_k = -\frac{\partial C}{\partial I}$.
We must accordingly add an expression $\pi_k(I - \varepsilon K)$ in our maximand H and Net National Product in consumption units will be

$$C + p_k(I - \varepsilon K) + p_\$ (F - R_B)^1, \text{ where } p_k = \frac{\pi_k}{U_1}.$$

---

1 We leave out $\dot{C}_m$ here and similarly we might have added and subtracted $p_\$ K_M$.

Figure 3



Corresponding to the dynamic equation for the new stock variable there will now be an equation for the price π_k or p_k (33) (assume $\beta=0$ in (15)):

$$\text{(33)} \qquad \dot{\pi}_k - q\pi_k = \varepsilon\pi_k - U'(C)\frac{\partial C}{\partial K}$$

or:

$$\frac{\dot{p}_k}{p_k} = q + (1-\alpha)\frac{\dot{C}}{C} - \rho$$

where $\rho = \frac{\partial C}{\partial K}\frac{1}{p_k} - \varepsilon$ = (net) rate of return on non-tradable capital.

Equation (33) is a dynamic equilibrium condition in the market for domestic capital goods which is well known from optimal growth theory for closed two sector economies (e.g., Uzawa [1962]). We note that the analogous condition for the price of foreign exchange is here retained without change even though we have a more general formulation. The consumption own rate of interest $[q + (1-\alpha)\frac{\dot{C}}{C}]$ must on the one hand equal the adjusted rate of return on foreign exchange assets $\left(\frac{\dot{p}_\$}{p_\$} + \bar{r}\right)$ and on the other hand that for domestic

capital ($\rho + \frac{\dot{p}_k}{p_k}$).

The fact that the previous part of the model remains intact does not, in the general case, imply that the solution will remain relatively simple. In general the four equation dynamic system will be interdependent, and the reduced form supply schedule of F (and similarly C and I) will be a function of both prices ($p_\$$, p_k) as well as the stock K. However, it may be useful to point out at least one simple case where the system is decomposable in one direction, in the sense that we can conditionally solve for the pair (B, $p_\$$) before we solve for the other pair (K, p_k). This can be illustrated by the case in which the non-tradable capital good only supplies services V(K) to final consumers (say housing or personal services) and is constructed only with the aid of a labour input (L_I). We must then add to equations (10)

$$p_k \frac{\partial I}{\partial L_I} = w,$$

and in equation (33) instead of $\partial C/\partial K$ we write $V'(K)/U_1$. Clearly some of the labour force must now be devoted to the production of I, and in this sense the labour available to production in the C and E sectors ($L - L_I$) is not independent of the solution of the K system. However, the nature of the general solution will remain the same.

Traditional growth theory can be said to have concentrated on an analysis in the C-I plane (see figure 3). Our own analysis of the previous sections has concentrated on the C-F plane. A fully fledged complete analysis would porbably place the economy somewhere on the three dimensional surface (typical point being U, say). The above illustration is only one

very simple example thereof. A complete general solution must await further research[1], but we believe enough has been said to suggest that under a broad set of conditions the addition of a domestic capital good could not alter the essence of the previous analysis.

2. Learning Effects

Our analysis has so far concentrated on a relatively simple equalisation of marginal rates of return in all forms of production and use of foreign exchange. To see that such equalisation may sometimes take more complex forms, consider the case of an economy in which there are externalities in the form of a learning process that takes place in one tradable good, call it E_1.[2] Let us further denote the accumulated sum of all past exports E_1 by D_1 and include it as one of the primary factors of production in the aggregate production possibility schedule. We now have $E_1 = \dot{D}_1$ and we get a static (34) as well as a dynamic (35) equation for an additional shadow price $\pi_{\$1}$ (in utility units) or $p_{\$1}$($= \frac{\pi_{\$1}}{U_1}$ in consumption units):

$$(34) \qquad p_{\$1} = -\frac{\partial C}{\partial E_1} - p_{\$} \qquad \text{or:} \quad -\frac{\partial C}{\partial E_1} = p_{\$1} + p_{\$} > p_{\$}$$

$$(35) \qquad \frac{\dot{p}_{\$1}}{p_{\$1}} = q + (1-\alpha)\frac{\dot{C}}{C} - \frac{1}{p_{\$1}}\frac{\partial C}{\partial D_1} = \frac{\dot{p}_{\$}}{p_{\$}} + \bar{r} - \frac{1}{p_{\$1}}\frac{\partial C}{\partial D_1}$$

According to (34) $p_{\$1}$ is the difference between the marginal rate of substitution of E_1 and C and the shadow exchange rate ($p_{\$}$). It is thus a measure of subsidy that must be allotted to E_1 over and above the

---

[1] The corresponding Jacobian becomes almost unmanageable, but an attempt is in process.

[2] In H it will appear in the pps, in the $\dot{B}$ constraint as well as a separate 'investment' good with price $\pi_{\$1}$, say

$$(\text{or } p_{\$1} = \frac{\pi_{\$1}}{U_1})$$

regular remuneration of foreign exchange. Equation (35) gives the rate of change of this subsidy over time. This will be larger or smaller than the rate of change of $p_\$$ according to whether the rate of return on experience

$$\frac{1}{p_{\$1}} \frac{\partial C}{\partial D_1}$$

is smaller or larger than the marginal social rate of interest on foreign assets. The former is probably more likely to be the case in practice. What this means is that there should both be an absolute subsidy and that its rate should be <u>increased</u> over time.[1] This is a natural way of incorporating 'infant industries' in our model.[2]

3. Autarchic economic development

We have so far taken it for granted that a country which faces a given capital market will behave optimally, in the sense that it will always borrow, if it pays to do so. Counter to what is often believed, it will not borrow to the point at which its internal own rate of return will equal the marginal interest cost, but rather have a combined borrowing cum trade policy through a rate of (real) devaluation that equals the difference between those two interest rates. With that in mind, it should borrow if it can and it pays to do so. Why do we historically observe some economies that develop without or hardly borrowing at all, or follow what is often

---

[1] It should be clear that whenever we say 'subsidy to a tradable good' what is implied is either a subsidy on an export good, on the basis of value added, or effective tariff on an import to protect an import substitute.

[2] See Bruno, Dougherty and Fraenkel [1970] and Bruno [1970], for a simple linear model formulation in an empirical context.

termed the 'Russian' model of autarchic development?

Other than coming up with arguments of non-optimality there could essentially be two ways of rationalizing such behavior within the confines of our model. One is to suggest the case in which the level of $\bar{r}$ that a particular country is facing at low (positive) levels of B to be so high in relation to its internal rate of profit that it will not borrow.[1] A more reasonable explanation, however, would be in terms of a modified social welfare function that includes B as a separate disutility item. Suppose we subtract from U a function v(B) $(v'(B) > 0,\ v''(B) \gtreqless 0)$, which stands for 'fear of colonialism', 'loss of political independence,' etc., over and above the straight interest rate cost involved. The implication in terms of the model would be to augment $\bar{r}$ in equation (8') by

$$\frac{v'(B)}{\pi_\$}$$

and thus make for higher revealed social cost of borrowing, lower B, higher $p_\$(0)$ and lower $\frac{\dot{p}_\$}{p_\$}$. It is hard to give this notion quantitative measure but there is no doubt that it exists and does often affect governments' decisions in practice.

4. Consumption constraints

We have so far assumed that labour is paid its marginal product and that the economy should have whatever consumption level was dictated by the

---

[1] In terms of the dynamic equation for $p_\$$ this would also mean that it has to set a very high initial level of $p_\$$ from the start and keep it there. This may explain what often looks like 'excessive' import substitution, but would not explain an anti-export bias that often goes with it.

particular optimal trade and growth path indicated. Since we have assumed our production functions to be neoclassical, w will never in fact be zero in this model. However, once we think of functions with very low elasticities of substitution in surplus labour economies, the resulting w or $(C_d + p_\$ C_m)/L$ might turn out too low to be realistic from any political let alone human point of view. A more realistic treatment of the labour market in such economies, as suggested by a number of authors[1], is to give explicit recognition to a minimum consumption (or maximum savings) constraint. One appropriate way to modify our treatment of the labour and consumption markets in the two-sector model might be to add constraints of the form:

$$(36) \qquad C_d - (c_1 L_C + c_2 L_E) \geqq 0$$

$$C_m - (m_1 L_C + m_2 L_E) \geqq 0 \qquad (c_i, m_i \geqq 0)$$

One now has to work out the new equilibrium conditions with a modified maximand that includes the two constraints (36) with lagrangian multipliers π_d and π_m, respectively. Proceeding just as before we now find that the utility price of domestic consumption good becomes $U_1+\pi_d$ and for the importable consumption good we have $U_2 + \pi_m = \pi_\$$.

Thus, consumption of the good must be driven up to a point at which the marginal utility, as ordinarily defined, is below the exchange rate. In other words, the import must be <u>subsidised</u> to the consumer, as compared to other tradable goods. Using U_1 as numeraire as before and denoting

---

1 See Marglin [1967] or the application to project evaluation criteria by Little and Mirrlees [1969].

$p_d = \frac{\pi_d}{U_1}$, $p_m = \frac{\pi_m}{U_1}$, the equilibrium conditions for the labour market (10) now become:

$$(37) \qquad \frac{\partial C}{\partial L_C} = w + (p_d c_1 + p_m m_1)$$

$$p_\$ \frac{\partial E}{\partial L_E} = w + (p_d c_2 + p_m m_2)$$

We may now have w = 0 in which case one (or both) of the constraints (36) takes over the determination of relative labour inputs instead of the original labour supply constraint.[1] At any rate the shadow wage rate in either sector may now be different from w and from each other, depending on the institutional constraints (e.g. union strength) in the two sectors. In terms of our original analysis what is implied is a more constrained transformation frontier (at each point in time) where this frontier now incorporates both production as well as institutional constraints. None of this, however, alters the basic dynamic structure of the system.[2]

5. Other trade 'distortions'

It should be clear from the former discussion that there is room for possible further refinements in the direction of differential exchange rates (i.e. taxes or subsidies) for different goods, depending on whatever 'second best' situations might arise. E.g., a country may have inherited a quota system on some key imports which it cannot feasibly alter from a political point of view (this would be a case of a ceiling on imports of a good,

---

[1] Obviously except by fluke we can never have all of the triple w, p_d, p_m different from zero. Where w = 0, however, one ought to bring in also the minimum consumption of the unemployed $(L - L_C - L_E)$.

[2] One could, of course, affect that too by specifying the c_i or m_i as time functions.

rather than a floor). Alternatively, the taxability of different goods may be different (e.g. it may be easier to tax imported goods than domestically produced ones). This is a factor that could be incorporated in the model once we bring tax functions into the picture. Since, however, there seems nothing basically new that would come out in the present context, we don't pursue the matter any further here. Suffice it to stress that whenever one talks of an equilibrium exchange rate or of its optimal movement over time, the present framework should serve as an indication that such equilibrium rates are to be meant in a <u>constrained</u> optimisation sense.

VI. EMPIRICAL RELEVANCE AND CONCLUDING REMARKS

The general equilibrium framework suggested here seems a useful means of characterizing both the growth paths of some key quantity and price variables in an open economy as well laying out the various interdependencies that must exist between them. It is important to quantify the relationship between the real wage, the current account gap, the exchange rate, their movement over time and the way the latter tie up with the various interest rate concepts that often get confused in discussions of factor pricing and public investment criteria. Thus one should distinguish between the rate of return on (domestic) capital, the rate of return on foreign (tradable) assets and the consumption own rate (or social discount rate). They all clear different markets but obviously hang together through various arbitrage mechanisms involving changes in relative asset prices. The relevance to the choice of cirteria as well as shadow prices for public investment in an open economy should be clear. Elsewhere [1970] we have indicated the use of simplified models of this kind in the context of the theory of dynamic comparative advantage.[1] Finally, as far as decision models go this type of framework also seems to provide ways of improvement on the rather simple minded two-gap analytical representations of development in an open economy.

Does this type of analysis also have some descriptive content for the actual behavior of economies? Let us start with a relatively less important

---

[1] Basically what is involved is defining comparative advantage in terms of the position of an activity (or industry) on the economy's production possibility curve, at different points in time, in relation to the equilibrium points.

point. Discussions of foreign aid and foreign capital inflow usually center around the investment promoting role of such aid. This bias comes out clearest in the way we break down the finance of investment in our national accounts into domestic and foreign savings, presuming them to be complementary factors. Empirical work recently done by Weisskopf [1970] and others points to significantly lower savings rates for developing countries that are, ceteris paribus, higher recipients of aid. In the context of a model of the kind outlined here such result would come as no surprise. It may, in fact, be optimal to use foreign debts not only for the finance of imports of capital goods but make for smoother consumption paths. What about more general characteristics of development and structural change?

The optimal growth paradigm as usually set in the environment of a closed two sector economy pictures the economy monotonically accumulating a physical asset,less capital-intensive than the consumption good, whose rate of return and relative demand price must systematically fall over time as long run bliss is approached. I know of no economy or government that does in fact manipulate the price of the capital good in this way, nor do I know of any statistics that would substantiate such systematic movements over time let alone the required capital intensities.

We have seen that in an open economy there is a more important relative price, the exchange rate, which one would also expect to be relevant to the price of <u>tradable</u> capital goods. Our theory would, under normal empirical circumstances, predict an <u>increase</u> rather than a decrease of this price over time.[1]

---

[1] Since most investments in real life are a combination of both tradable and non-tradable assets it is hard to make any apriori predictions on the price of composite investment.

Moreover the exchange rate, unlike the price of non-tradable goods, is a variable that most governments _do_ manipulate. Do governments in fact follow such rules? I am sure no one in my own government has ever studied, let alone used, optimal growth theory for the formulation of foreign exchange policy and may be it is a good thing they have not. Yet like the stylized entrepreneur in the theory of the firm, they appear to behave _as if_ they followed some such rules.

In an empirical context the 'exchange rate' as here defined should be interpreted to be measured by what is sometimes called the _effective exchange rate_, i.e. the official exchange rate plus tariffs in the case of imports and including subsidies, in the case of exports, ideally measured on the basis of net value added in both cases and suitably deflated by a domestic price level. Michaely [1968] has already shown that in the 1950's in Israel the various (gross) exchange rates did in fact show such consistent rise over time. We have recently looked ourselves into longer term series for the effective subsidy rate on Israeli manufacturing exports during 1950-1969 (deflated by the wholesale price of domestic manufactures) and found a consistent rise throughout most of the period, with $p_{\$}$ thus measured almost doubling (!) over 20 years. There was a more rapid increase in the early years 1950-58, an average 1% drop during the high inflation years (1958-63) and an annual average increase of 4.5% during the last six years. There is hardly any doubt, and various econometric studies[1] have substantiated it, that this use of the effective exchange rate for export promotion provides at least part of the explanation for the phenomenal increase of Israel's manufacturing exports. These rose from a level of $9 million

1 See M. Evans [1968]. More recently my colleague Nadav Halevi has been conducting a more detailed study on this.

in 1950 to $350 million in 1970 (over half of total commodity exports).

Until recently Israel had been obtaining its capital inflow at a marginal cost of around 6-8%. With a $\dot{p}_\$/p_\$$ of 4.5% could we infer that the consumption own rate of return has been 10.5-12.5%? Internal real rates of return seem to accord reasonably well with such orders of magnitude. However, we may by now be taking our theoretical model too seriously.

I have tried in vain to search for any relevant empirical work for other countries. For some reason this important empirical question of what happens to <u>real</u> exchange rates over time has not yet been studied systematically. The only remotely relevant empirical study I have come to know of [1] is a paper by Yaeger [1958], in which he uses official exchange rates and domestic price data for the years 1938 and 1957 to try and rehabilitate the purchasing power parity theory.[2]

Clearly neither the Israeli government nor presumably any other government would ever set the price $p_\$$ on the basis of an exact specification of a horizon T, loan schedule and complete knowledge of all future production functions. Yet our discussion of figure 2 has shown that a general pattern of the kind discussed might also be followed if there is a fair amount of uncertainty involved, as long as the economy does not cross on the other side of the critical Q_1SQ_2 path. The kind of analysis that is being called for now is a more detailed and maybe more realistic specification of the short run equilibrating process that an economy undertakes when

---

[1] Thanks to Charles Kindleberger.

[2] Counter to the conclusion reached in that paper I find his own data on the whole point to systematic changes in exchanges rates which are <u>not</u> explained by movements in domestic price levels. In any case, however, both the time period as well as the sample of countries should now be greatly extended.

it find out that it has, so to say, chosen the wrong initial $p_\$$ e.g., when detecting that B has run off along the curve D_1 say.

But there may be more fundamental issues worth pursuing. We have all the time implicitly assumed that the government can set the <u>real</u> exchange rate, whereas what governments usually do is to set the <u>nominal</u> exchange rate (or multiple rates in the broad sense, including taxes and subsidies) and affect the domestic price level independently through their monetary, fiscal and income policies. Obsiously all of these should be analysed simultaneously within one framework. Finally, a point more directly related to the present model - in many actual cases our specification of the foreign loan market would be over simplified - supply schedules may have kinks in them and what is worse - often the amount of concessionary aid forthcoming is subject to unknown fluctuations. We therefore end with the inevitable sentence - there is room for more...

REFERENCES

Adelman, I. and E. Thorbecke (eds.): The Theory and Design of Economic Development, Johns Hopkins, 1966.

Arrow, K.J., and M. Kurz: Public Investment, the Rate of Return and Optimal Fiscal Policy, 1970.

Bardhan, P.: Optimum Foreign Borrowing" in K. Shell (ed.), Essays in the Theory of Optimal Economic Growth, M.I.T., 1967.

Bardhan, P.: "Optimum Growth and Allocation of Foreign Exchange", Working Paper no. 36, Department of Economics, M.I.T., January 1969.

Bruno, M.: "Development Policy and Dynamic Comparative Advantage", in The Technology Factor in International Trade, National Bureau of Economic Research, 1970.

Bruno, M., C. Dougherty and M. Fraenkel, "Dynamic Input-Output, Trade and Development", in Carter and Brody (eds.), see below (1970).

Carter, A. and A. Brody (eds.): Applications of Input-Output Analysis, North Holland, 1970.

Evans, M.: "An Econometric Model of Part of the Israel Economy", Discussion Paper no. 86, Department of Economics, University of Pennsylvania, 1968.

Fraenkel, M.: "Planning the Balance of Payments and Criteria for Foreign Borrowing", Paper submitted at Seventh Meeting of Senior Economic Advisors, ECE, 1969. (Bank of Israel, October, 1969).

Hamada, K.: "Optimal Capital Accumulation by an Economy Facing an International Capital Market," Journal of Political Economy, July-August 1969.

Little I.M.D., and J.A. Mirrlees: Manual of Industrial Project Analysis, Vol. II, OECD, 1969.

Marglin, S.: "The Rate of Interest and the Value of Capital with Unlimited Supplies of Labour", in Shell (ed.), op.cit.

Michaely, M.: The System of Effective Exchange Rates in Israel, Falk Institute, Jerusalem, 1968.

Oniki and H. Uzawa: "Patterns of Trade and Investment in a Dynamic Models of International Trade", Review of Economic Studies, 1965.

Ryder, H.: "Optimum Accumulation and Trade in an Open Economy of Moderate Size", in K. Shell (ed.), op. cit. (1967).

Teubal, M.: "A Trade and Development Model of an Economy with Intersectorial Relations", (mimeog.) Jerusalem 1970.

Uzawa, H.: "On a Two-Sector Model of Economic Growth", Review of Economic Studies, Vol. XXIX (1961-1962) and Vol. XXX (1962-63).

Yaeger, L.B., "A Rehabilitation of Purchasing-Power Parity", Journal of Political Economy, February 1958.

Weisskopf, T., "The Impact of Foreign Capital Inflow on Savings in Underdeveloped Countries," (mimeog.), Harvard, September 1970.